

NASA TECHNICAL NOTE



NASA TN D-5652

NASA TN D-5652

*Filed as;
N 7018963*

**SURVEY OF LITERATURE ON
FLOW CHARACTERISTICS OF
A SINGLE TURBULENT JET
IMPINGING ON A FLAT PLATE**

by James W. Gauntner, John N. B. Livingood, and Peter Hrycak*

*Lewis Research Center
Cleveland, Ohio*

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • FEBRUARY 1970

1. Report No. NASA TN D-5652	2. Government Accession No.	3. Recipient's Catalog No.	
4. Title and Subtitle SURVEY OF LITERATURE ON FLOW CHARACTERISTICS OF A SINGLE TURBULENT JET IMPINGING ON A FLAT PLATE		5. Report Date February 1970	
		6. Performing Organization Code	
7. Author(s) James W. Gauntner and John N. B. Livingood, Lewis Research Center; and Peter Hrycak, Newark College of Engineering		8. Performing Organization Report No. E-5203	
9. Performing Organization Name and Address Lewis Research Center National Aeronautics and Space Administration Cleveland, Ohio 44135		10. Work Unit No. 720-03	
		11. Contract or Grant No.	
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D. C. 20546		13. Type of Report and Period Covered Technical Note	
		14. Sponsoring Agency Code	
15. Supplementary Notes			
16. Abstract Flow characteristics of single jets impinging on flat surfaces have been studied by many investigators. The results of some of the numerous studies are summarized herein. Suggested methods for determining velocities and pressures on which to base heat-transfer correlations for use in impingement cooling design are presented.			
17. Key Words (Suggested by Author(s)) Literature survey Impinging jet Flat plate Flow characteristics		18. Distribution Statement Unclassified - unlimited	
19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of Pages 45	22. Price* \$3.00

*For sale by the Clearinghouse for Federal Scientific and Technical Information
Springfield, Virginia 22151

CONTENTS

	Page
SUMMARY	1
INTRODUCTION	1
DESCRIPTION OF FLOW REGIONS	2
GENERAL FEATURES OF JETS	3
REGION OF FLOW ESTABLISHMENT (REGION I)	4
Velocity Profile	5
Potential Core Length	9
Centerline Turbulence Distribution	14
Pressure Distribution	15
Secondary Effects	15
REGION OF ESTABLISHED FLOW (REGION II)	15
Centerline Velocity Decay	15
Velocity Profile	16
Turbulence Distribution Along Centerline	22
Pressure Variation Along Jet Centerline	23
Static Pressure Profiles	25
Spread of Free Jet	26
REGION OF DEFLECTION (REGION III)	26
Velocity Distribution	26
Pressure Distribution	28
WALL JET REGION (REGION IV)	29
Velocity Decay Along Plate	29
Velocity Profile Through Wall Jet	30
Spread of Wall Jet	34
POSSIBLE APPLICATIONS FOR USE IN HEAT-TRANSFER CORRELATIONS	34
CONCLUDING REMARKS	37
APPENDIX - SYMBOLS	39
REFERENCES	41

SURVEY OF LITERATURE ON FLOW CHARACTERISTICS OF A SINGLE TURBULENT JET IMPINGING ON A FLAT PLATE

by James W. Gauntner, John N. B. Livingood, and Peter Hrycak*

Lewis Research Center

SUMMARY

The results of an extensive survey of the flow characteristics of a single jet impinging on a flat surface are presented herein. The flow field consists of four distinct regions, each of which is discussed in detail. Methods for predicting the spread of the jet, and velocity and pressure profiles in the various regions, are presented.

Comparisons between theory and experimental data are presented for velocity distributions in the various flow regions. Experimental data for velocity fluctuations due to turbulence are also shown. Some suggested velocities on which to base heat-transfer correlations for use in impingement cooling design and some areas requiring further investigations are also discussed.

INTRODUCTION

A promising method for cooling turbine vanes or blades is to impinge cool air on the internal surfaces of the vanes or blades. Although numerous investigators have studied the impingement of air jets on flat plates, heat-transfer results differ so widely that no adequate heat-transfer correlation for design use is available. It is for this reason that the NASA Lewis Research Center initiated an impingement cooling program. The ultimate goals of this program are to develop design heat-transfer correlations to apply to cylindrically shaped leading edges of vanes or blades and for impingement cooling along the vane or blade suction and pressure surfaces. The initial phase is limited to impingement on a flat plate. Such results may be applicable for cooling blade suction and pressure surfaces.

Development of adequate heat-transfer correlations depends on a knowledge of flow characteristics. A number of investigators have studied the flow phenomena of free jets

*Professor of Mechanical Engineering, Newark College of Engineering, Newark, N. J.

issuing from both circular holes and slots. Some have also investigated jets impinging on flat surfaces. Our purpose herein is to summarize many of these previous studies and to include derivations of useful equations for determining pressures and velocities in the flow field. This report is based, in part, on a survey of literature on flow and heat transfer caused by impingement on a flat plate completed under NASA Grant NGR 31-009-004 by Professor Peter Hrycak of the Newark College of Engineering. This report is restricted to the study of the flow field of a single jet impinging on a smooth flat plate.

DESCRIPTION OF FLOW REGIONS

The various flow regions formed by a jet impinging on a solid surface have been discussed by previous investigators (e. g., refs. 1 and 2). In the case of an impinging jet, four distinct flow regions can be characterized. These are shown in figure 1 (symbols defined in the appendix) and are described as follows:

(1) Region I is the region of flow establishment. It extends from the nozzle exit to the apex of the potential core. The so-called potential core is the central portion of the flow in which the velocity remains constant and equal to the velocity at the nozzle exit.

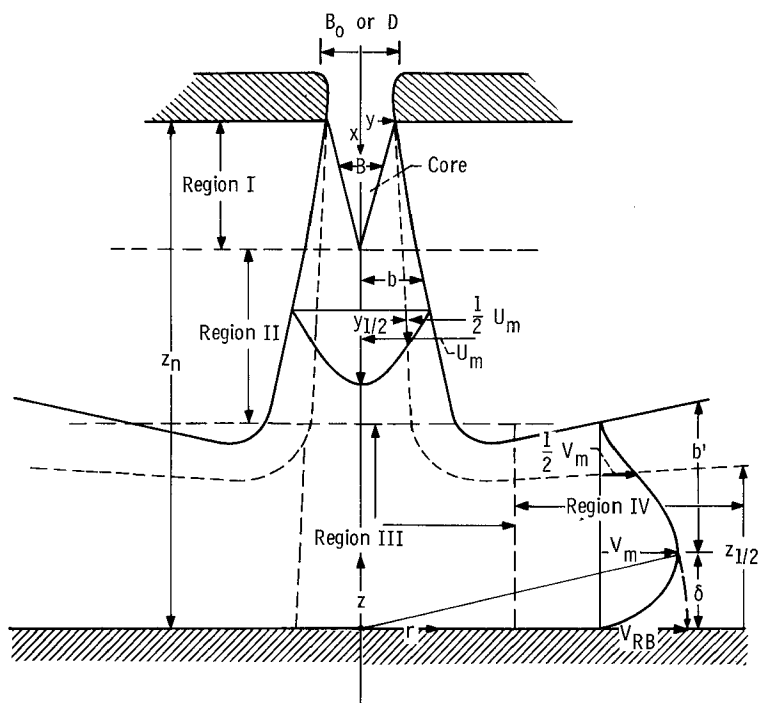


Figure 1. - Characteristic regions in impinging jet flow.

(2) Region II is a region of established flow in the direction of the jet beyond the apex of the potential core; it is characterized by a dissipation of the centerline jet velocity and by a spreading of the jet in the transverse direction.

(3) Region III is that region in which the jet is deflected from the axial direction.

(4) Region IV is known as the wall jet region, where the directed flow increases in thickness as the boundary layer builds up along the solid surface.

GENERAL FEATURES OF JETS

Of fundamental importance in the study of impinging jets (or any jet) is the distinction between laminar and turbulent jets. Reference 3 states that the critical Reynolds number Re_0 (based on nozzle diameter and nozzle exit velocity) which distinguishes laminar jets from turbulent jets is about 1000. Reference 4 reports four characteristic jet patterns for free jets, namely,

(1) The dissipated laminar jet, $Re_0 < 300$. In this case, the viscous forces are large compared to the inertial forces, and the jet diffuses rapidly into the surrounding fluid.

(2) Fully laminar jets, $300 < Re_0 < 1000$. In this case, there is no noticeable diffusion of the jet into the surrounding fluid.

(3) A transition or semiturbulent jet, $1000 < Re_0 < 3000$.

(4) A fully turbulent jet, $Re_0 > 3000$.

These results are confirmed in reference 5, which also states that jets are turbulent for $Re_0 > 3000$. Reference 6, however, refers to a turbulent jet as having Re_0 greater than 14 000. The reason for this difference in Re_0 at which turbulence begins could not be resolved from information contained in the references. The fact that differences have been obtained indicates that further work is required to determine the factors affecting this characteristic. From a heat-transfer standpoint, the determination of Re_0 at which a jet becomes turbulent is vital because applications of cooling with impinging jets involve Re_0 as low as 3000.

With regard to different regions of the jet, reference 7 states that two different modes of heat transfer exist. The first mode applies to the potential core region, where turbulent fluctuations cause the variation in the heat-transfer coefficient. The second mode is for the region of established flow, where both turbulence and velocity cause the variation.

The preceding discussion relating jet pattern to Re_0 range dealt only with jets issuing from circular openings. However, it appears to apply also to jets issuing from slots. Reference 8 indicates that slot jets are turbulent for $Re_0 = 2000$ (here Re_0 is based on the slot width, whereas for circular jets it is based on jet diameter).

The disturbance created by a solid wall on an impinging jet is propagated upstream at a rate equal to the difference between the sonic velocity and the fluid velocity. The wall effect would therefore diminish with increasing Mach number and would finally disappear when the Mach number equaled 1 and the disturbance created by the wall would be unable to propagate upstream.

In a theoretical study reported by Levey (ref. 9) the effect of the impingement surface on the jet was not felt by the jet as close as only 1 slot width away from the surface; this study was for an idealized case (incompressible flow) with no account for viscous effects or turbulent mixing. An experimental study reported in reference 10 confirms the theoretical results; no surface effect on the jet was observed beyond about 2 jet diameters from the plate. Consequently, an impinging jet can be considered to behave like a free jet except in the immediate vicinity of the impingement surface.

REGION OF FLOW ESTABLISHMENT (REGION I)

Near the nozzle opening, a mixing zone originates and turbulent exchange of heat and mass takes place. The mixing zone grows in width in the downstream direction of the jet, leaving a potential core wherein fluid properties and velocity are relatively constant. Eventually, the core is dissipated as a result of the mixing action.

According to Schlichting (ref. 11), by a judicious selection of the origin in the coordinate system, the half width b (see fig. 1) of the free jet is directly proportional to the distance along the jet centerline; that is,

$$b = \text{Constant } x \quad (1)$$

This equation may be used as an approximation for the underdeveloped jet. According to reference 12, the pressure in region I is virtually constant and equal to the pressure in the surrounding fluid. Hence, the total momentum per unit time should be the same for all cross sections normal to the jet; that is,

$$J = \int U \, dM = \rho \int_0^{\infty} U^2 \, dA = \text{Constant} \quad (2)$$

Velocity Profile

Experimental data of reference 13 suggest that the sole force producing the deceleration of the jet and the acceleration of the surrounding fluid is the tangential shear within the mixing region. An approximate analysis of the mean velocity distribution was presented in reference 13 and is summarized in this section (see fig. 2).

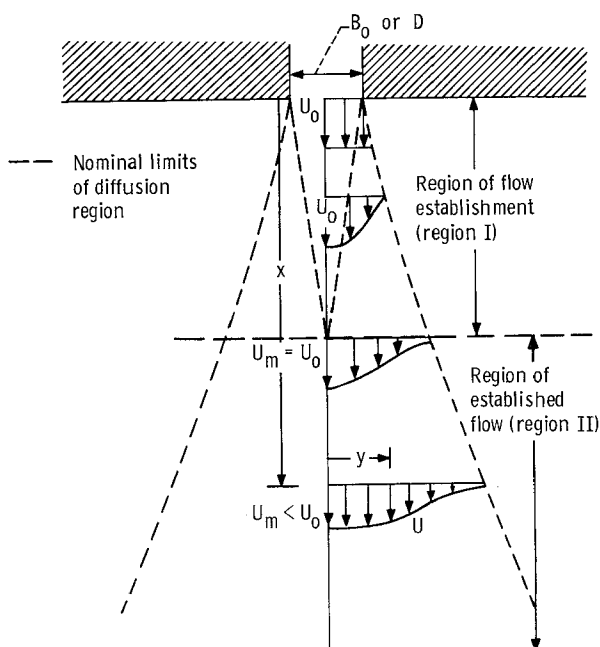


Figure 2. - Schematic representation of jet diffusion.

The volumetric flow rate Q past successive normal sections may be written as

$$Q = \int_0^{\infty} U \, dA \quad (3)$$

Since Q varies with longitudinal distance x , its ratio to Q_0 (at nozzle exit) is

$$\frac{Q}{Q_0} = \frac{\int_0^{\infty} U \, dA}{U_0 A_0} \quad (4)$$

where A_0 is nozzle exit area. Also, since momentum flux J may be expressed as the integral of the volume rate of flow $U \, dA$ times the longitudinal component of momentum per unit volume ρU ,

$$\frac{J}{J_0} = \frac{\int_0^\infty U^2 dA}{U_0^2 A_0} \quad (5)$$

Also, energy flux equals the integral of the local volume flux $U dA$ times the kinetic energy per unit volume $\rho(U^2/2)$,

$$\frac{E}{E_0} = \frac{\int_0^\infty U^3 dA}{U_0^3 A_0} \quad (6)$$

Since the tangential shear within the mixing zone causes the deceleration of the jet and the acceleration of the surrounding fluid and since this is an internal process, the momentum flux must be constant at every normal section; that is,

$$\frac{J}{J_0} = \frac{\int_0^\infty U^2 dA}{U_0^2 A_0} = 1 \quad (7)$$

If viscous action has no influence on the mixing process, the mean flow should be dynamically similar under all conditions. Thus, a similar velocity profile must characterize every section in the diffusion region. As a matter of fact, experimental data follow the general trend of the Gaussian distribution

$$\frac{U}{U_m} = \exp\left(-\frac{y^2}{2\sigma^2}\right) \quad (8)$$

where y is the transverse distance from the jet centerline or axis and σ is the standard or root-mean-square deviation (the value of y for which $U = 0.606 U_m$; see fig. 3).

This equation is merely a result of a curve fit and has no theoretical basis. The condition of dynamic similarity requires that at all cross sections, regardless of the efflux velocity,

$$\frac{\sigma}{x} = K_1 \quad (9)$$

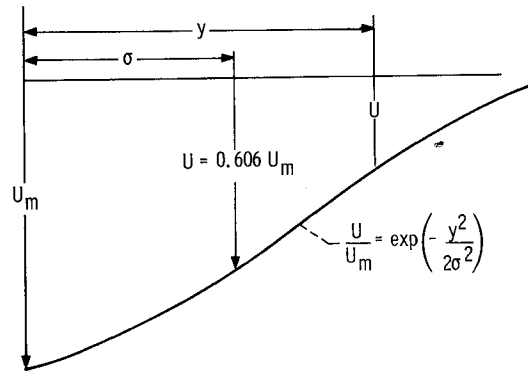


Figure 3. - Characteristics of normal probability curve.

for the slot jet and

$$\frac{\sigma}{x} = K_2 \quad (10)$$

for the circular jet (i. e., the angle of jet diffusion is a constant). Values of K_1 and K_2 must be determined experimentally. In real jets, the angle of jet diffusion is not necessarily a constant immediately downstream from the nozzle. In reference 13, σ/x proves to be different for both the developed jet (region II) and the developing jet (region I). Still an attempt has been made in reference 13 to treat the jet as if the same experimental constant were applicable for both regions.

Using equations (8) and (10) in evaluating equation (7) yields an expression relating the average nozzle exit velocity to the axial velocity at any point downstream of the potential core for the circular jet

$$\frac{J}{J_0} = \frac{U_m^2 K_2^2 x^2 \pi}{U_0^2 A_0} = 1 \quad (11)$$

This equation can then be evaluated at the point where the potential core region ends (i. e., where x equals x_c and U_m still equals U_0). The appropriate value of A_0 may also be substituted into equation (11). Similar substitutions can be made for the slot jet, where the momentum is put on a per unit length basis. These substitutions yield

$$\frac{x_c}{B_0} = \frac{1}{\sqrt{\pi} K_1} \quad (12)$$

for the two dimensional jet and

$$\frac{x_c}{D} = \frac{1}{2K_2} \quad (13)$$

for the circular jet. The nominal inner boundary of the diffusion region satisfies the equation

or

$$\left. \begin{aligned} \frac{B}{B_0} &= 1 - \frac{\sqrt{\pi} K_1 x}{B_0} \\ \frac{B}{D} &= 1 - \frac{2K_2 x}{D} \end{aligned} \right\} \quad (14)$$

where B is the width of the potential core at a distance x from the nozzle. The velocity distribution in the diffusion zone of region I, obtained by combining equations (8) to (10) and (14), becomes

$$\frac{U}{U_0} = \exp \left[- \frac{\left(y + \frac{\sqrt{\pi} K_1 x}{2} - \frac{B_0}{2} \right)^2}{2(K_1 x)^2} \right] \quad (15)$$

for a slot of half width $B_0/2$ and

$$\frac{U}{U_0} = \exp \left[- \frac{\left(y + K_2 x - \frac{D}{2} \right)^2}{2(K_2 x)^2} \right] \quad (16)$$

for a circular jet. Experimental data are compared in figures 10 and 16 of reference 13 with results calculated from equations (15) and (16), respectively. Satisfactory agreement between equation (15) and experiment is shown in figure 10 of reference 13. Experimental data shown in figure 16 of reference 13 deviate considerably from equation (16). This is due to the fact that reference 13 attempted to use a single value of σ/x for both the region of flow establishment and the region of established flow. A better comparison would result by using the value of σ/x associated with region I.

Potential Core Length

It was previously stated that region I extends from the nozzle to the apex of the potential core. The potential core length considered in this report is sometimes called the nominal potential core length and its determination requires a knowledge of the centerline velocity decay in region II. For that reason, some material which in reality belongs in region II is presented under region I and then not repeated in region II. The actual potential core length, which corresponds to the spacing between the virtual origin of the jet and the point where the jet centerline velocity begins to decrease, is not referred to herein. If equation (11) is solved for the ratio of the centerline velocity to the average nozzle velocity for the circular jet, and a similar equation is solved for the slot jet, the following expressions result:

$$\frac{U_m}{U_o} = \frac{\sqrt{\frac{1}{\sqrt{\pi} K_1}}}{\sqrt{\frac{x}{B_o}}} \quad (17)$$

for the slot jet and

$$\frac{U_m}{U_o} = \frac{\frac{1}{2K_2}}{\frac{x}{D}} \quad (18)$$

for the circular jet. These equations may be reexpressed, using equations (12) and (13), as

$$\frac{U_m}{U_o} = \frac{\sqrt{\frac{x_c}{B_o}}}{\sqrt{\frac{x}{B_o}}} = \frac{\sqrt{C_1}}{\sqrt{\frac{x}{B_o}}} \quad (17)$$

$$\frac{U_m}{U_o} = \frac{\frac{x_c}{D}}{\frac{x}{D}} = \frac{C_2}{\frac{x}{D}} \quad (18)$$

The nondimensional potential core lengths C_1 and C_2 in equations (17) and (18) are for the slot jet and the circular jet, respectively.

Although equations (17) and (18) are only valid beyond the potential core (region I), they may be used to experimentally determine the length of the potential core in the following way. Log-log plots of the measured centerline velocity ratio U_m/U_o against x/B_o (or x/D) yield lines with slopes $-1/2$ or -1 , respectively, for the slot jet and the circular jet. Extension of these lines to intersect $U_m/U_o = 1$ then gives the desired length of the potential core, C_1 for the slot jet or C_2 for the circular jet.

Schlichting (ref. 11) presents the following equation for the velocity distribution in the axial direction of a developed laminar circular jet, from which an analytical expression for the nominal potential core length can be obtained:

$$U = \frac{3 K}{8\pi \nu x} \frac{1}{\left(1 + \frac{1}{4} \xi^2\right)^2} \quad (19)$$

where the kinematic momentum flux

$$K = 2\pi \int_0^{\infty} U^2 y \, dy$$

and

$$\xi = \sqrt{\frac{3K}{16\pi \nu x}} \frac{y}{x} \quad (20)$$

Along the jet axis, $\xi = 0$ and the velocity distribution along the axis reduces to

$$U_m = \frac{3 K}{8\pi \nu x} \quad (21)$$

and

$$K = \frac{\pi U_o^2 D^2}{4} \quad (22)$$

Therefore,

$$U_m = \frac{3}{8\pi} \frac{1}{\nu x} \frac{\pi U_o^2 D^2}{4}$$

or

$$\frac{U_m}{U_o} = \frac{3}{32} \text{Re}_o \left(\frac{x}{D} \right)^{-1} \quad (23)$$

This last equation is similar to equation (18) and shows that the potential core length for laminar flow (the coefficient of $(x/D)^{-1}$, which in this case is $(3/32)\text{Re}_o$) is a function of the nozzle Reynolds number.

For turbulent flow, the axial velocity is represented by equation (19) if, according to reference 11 the laminar flow kinematic viscosity ν is replaced by the virtual kinematic viscosity ϵ for turbulent flow, where ϵ has been evaluated as

$$\epsilon = 0.0256 U y_{1/2} \quad (24)$$

It is shown that $y \propto x$ and $U \propto x^{-1}$ (for similar flow). Hence, it is possible to express ϵ as

$$\epsilon = \nu \gamma \text{Re}_o \quad (25)$$

where γ is a proportionality factor. The resulting equation for the jet centerline axial velocity for the turbulent flow case then becomes

or

$$\left. \begin{aligned} U_m &= \frac{3}{32} \frac{U_o^2 D^2}{\gamma \nu \text{Re}_o x} \\ \frac{U_m}{U_o} &= \frac{3}{32\gamma} \left(\frac{x}{D} \right)^{-1} \end{aligned} \right\} \quad (26)$$

Hence, for turbulent flow, the length of the potential core is $(3/32)\gamma$ and is independent of the nozzle Reynolds number as long as flow remains similar and the definition of ϵ holds.

Another expression for potential core length of a turbulent circular jet may be obtained from the results of Tollmien. He analyzed the flow in the downstream part of a turbulent submerged axially symmetric jet. His solution may be used to get the expression shown in reference 12

$$\frac{U_m}{U_o} = \frac{0.48 D}{ax} \quad (27)$$

This result was obtained with the use of a similarity variable

$$\varphi = \frac{y}{ax} \quad (28)$$

and numerical integration. From table I in reference 14, when $U = U_m/2$ and $y = y_{1/2}$, $\varphi = 1.23$. From experiments described in reference 12,

$$y_{1/2} = 0.097 x \quad (29)$$

Hence, from equations (27) to (29),

$$\frac{U_m}{U_o} = \frac{0.48D\varphi}{y_{1/2}} \approx 6.1 \frac{D}{x} \quad (30)$$

Comparison of this result with equation (18) shows $C_2 \approx 6.1$ for a circular jet.

Unfortunately, values of core lengths differ for investigations carried out by different experimenters. The amount of divergence which different observers report generally exceeds the expected experimental error and is more than likely explained by the effect of the scale and the intensity of turbulence within the jet. Also the actual velocity distribution at the nozzle seems to be a factor. The following two paragraphs give the results of some of the previous investigators and some information concerning their experiments.

For slot jets, Albertson (ref. 13) states that the potential core extends 5.2 slot widths. He investigated air flowing through slots of widths 1/4, 1/16, and 1/32 inch (0.635, 0.159, and 0.079 cm) for a range of Reynolds numbers

$$1.5 \times 10^3 < Re_o < 7.2 \times 10^4$$

Schauer and Eustis (ref. 15) investigated a 1/2-inch (1.27-cm) slot; air impinged on a plate located from 10 to 40 slot widths from the nozzle. The values of Re_o ranged from 2×10^4 to 5.3×10^4 . The core length of about 5.5 slot widths used in the analysis presented in reference 15 was obtained by averaging two similar values obtained from the data of van der Hegge Zijnen (ref. 16). Reference 16 considered slot widths of about 0.197 and 0.394 inch (0.5 and 1.0 cm) and a value of Re_o of about 1.3×10^4 . Reference 17 reports results for various-sized rectangular, square, elliptical, and triangular slots, for $3.6 \times 10^3 < Re_o < 8.8 \times 10^4$ and arrived at an average value of potential core length of about 7 slot widths.

For circular air jets, reference 13 reported a value for C_2 of 6.2 after testing nozzles 1/4, 1/2, and 1 inch (0.635, 1.27, and 2.54 cm) in diameter for values of Re_o from 2.2×10^4 to 8.7×10^4 . Pai (ref. 18) reported a value for C_2 of 6.5. From the results of Tani and Komatsu (ref. 10), the corresponding value for C_2 is about 4.8; this result was obtained from data for air flowing through nozzles with diameters of about 2.05 and 3.27 inches (5.2 and 8.3 cm) and impinging on a plate 2 to 15 diameters from the nozzle. Poreh and Cermak (ref. 1) experimented with water flowing through nozzles 3/4, 1, and $1\frac{1}{2}$ inches (1.905, 2.54, and 3.81 cm) in diameter impinging on a plate located from 8 to 32 nozzle diameters from the nozzle; they report a value for C_2 of 7.7. From data shown on figure 2 of reference 1, the local jet centerline velocity can be observed to be about 10 percent higher than the average nozzle exit velocity. If the potential core length of reference 1 were to be reexpressed in terms of the maximum nozzle exit velocity, the numerical value would be more comparable to values obtained by other investigators.

In an appendix to reference 13, Baines examined the results of Albertson more closely and concluded that the Reynolds number has an effect on the potential core length. He recommended values for C_2 of 5 for $Re_o = 1.4 \times 10^4$ and 7 for $Re_o = 10^5$. Abramovich (ref. 12), however, asserts that the value of C_2 does not vary with Reynolds number but does vary with the velocity profile. However, since the velocity profile in turbulent flow may be considered to be a weak function of the Reynolds number, to some extent C_2 could vary with Re_o as Baines suggested. However, theory based on Prandtl's second hypothesis shows that, for developed turbulent flow, the potential core length is independent of the Reynolds number (eq. (26)).

To summarize, it has been noted that, for turbulent flow, potential core lengths varying from about 4.7 to 7.7 have been obtained by different investigators. Moreover, some investigators state that core length is dependent on Re_o , while others state that core length is independent of Re_o . This latter view is supported by equation (26) of this re-

port. Since, from a heat-transfer standpoint core length is important, it appears that additional work in this area is required.

Centerline Turbulence Distribution

Investigations have shown (refs. 6 and 7) that the local heat-transfer coefficient at the stagnation point on the impingement plate reaches a maximum value at the apex of the potential core. Since the centerline velocity is constant in the potential core, the observed increase in heat-transfer coefficient must be due to the effects of turbulence. Reference 19 shows that the turbulence level on the jet centerline begins to rise at an axial distance of only 1 diameter from the nozzle mouth; this is shown in figure 4, taken from reference 19, for both a 1-inch (2.54-cm) and a 3-inch (7.62-cm) heated circular jet. Nozzle size was also shown to affect turbulence and heat transfer. Consequently, any reference velocities that may be used or applied in heat-transfer analysis or correlation may need to be modified to include the effects of turbulence.

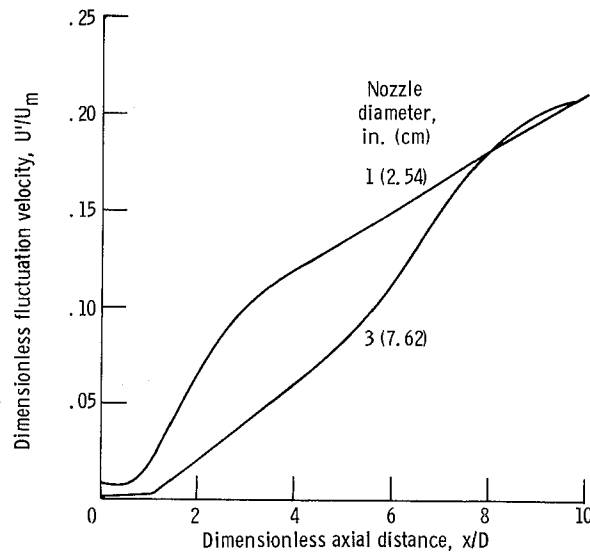


Figure 4. - Turbulence distribution along jet centerline. (Data from ref. 19.)

Pressure Distribution

The study of turbulent free jets by use of the boundary-layer theory assumes negligible pressure variation in the flow direction. However, reference 20 reports observation of a radial pressure gradient at a value of $x/B_0 = 2$ (within the potential core) with the maximum pressure at the centerline higher than ambient. A minimum pressure lower than ambient was observed away from the centerline of the jet. Reference 20 also reports a static pressure ridge, higher than ambient and decaying rapidly with x , in the potential core. The ridge is attributed to the persistence of the pressure distribution created within the nozzle. Outside the core in region I, all measured static pressures were below the ambient pressure.

From available theory and data, it appears that the effect of a pressure gradient in the potential core zone is relatively unimportant with respect to flow development, but it seems to warrant an additional experimental investigation.

Secondary Effects

The preceding discussions were limited to a study of the length of the potential core and the variation of velocity and pressure in the diffusion region. Secondary effects produced by the impingement surface may also have some effect on the jet. These secondary effects are (1) a dynamic instability effect, and (2) a blockage or interference effect. The former is characterized by a lateral oscillation of the complete jet flow field while the latter is distinguished by changes in core length and jet spreading rate.

REGION OF ESTABLISHED FLOW (REGION II)

Some of the analyses that were required to determine the potential core length and the velocity profiles for region I are only applicable to region II. These analyses are not repeated, but reference to the appropriate equations is made.

Centerline Velocity Decay

Based on discussion contained in reference 13, the centerline velocity decay for the slot jet is given by equation (17)

$$\frac{U_m}{U_o} = \sqrt{C_1} \left(\frac{x}{B_o} \right)^{-1/2} \quad (17)$$

and for the circular jet by equation (18)

$$\frac{U_m}{U_o} = C_2 \left(\frac{x}{D} \right)^{-1} \quad (18)$$

Many experimenters (e. g., Albertson et al. (ref. 13)) have investigated the centerline velocity decay, and their results have verified the general form of these equations.

Velocity Profile

According to Schlichting (ref. 11), problems in free turbulent flow are of a boundary-layer nature and can be studied with the aid of equations of boundary-layer type. For the two-dimensional, steady-state case, these equations are

$$U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} = \frac{1}{\rho} \frac{\partial \tau}{\partial y} \quad (31)$$

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0 \quad (32)$$

where τ denotes the turbulent shearing stress and where the pressure term has been dropped (to a first approximation, the pressure is assumed to be constant).

To obtain the velocity distribution from these equations, it is necessary to express τ in terms of parameters of the main flow. This may be done in three ways:

(1) Use of Prandtl's mixing length theory, where

$$\tau = -\overline{\rho uv} = \rho l^2 \left| \frac{\partial U}{\partial y} \right| \frac{\partial U}{\partial y} \quad (33)$$

(2) Extension of the mixing length theory, where

$$\tau = \rho l^2 \frac{\partial U}{\partial y} \sqrt{\left(\frac{\partial U}{\partial y}\right)^2 + l_1^2 \left(\frac{\partial^2 U}{\partial y^2}\right)^2} \quad (34)$$

and l and l_1 are to be regarded as purely local functions

(3) Prandtl's second hypothesis

$$\tau = \rho \epsilon \frac{\partial U}{\partial y} = \rho \chi_1 b (U_{\max} - U_{\min}) \frac{\partial U}{\partial y} \quad (35)$$

where b is the half width of the mixing zone (from centerline to outer jet boundary) and χ_1 is an empirical constant. From this, $\epsilon = \chi_1 b (U_{\max} - U_{\min})$ is the virtual kinematic viscosity, assumed to be constant over the whole width and previously used in equation (24).

Tollmien (ref. 14) obtained the velocity distribution by solving the system of equations (31) to (33); the use of equation (33) involves the application of Prandtl's mixing length theory for evaluating the shear stress.

Görtler (ref. 21) solved the system of equations (31), (32), and (35); this solution involved the use of Prandtl's second hypothesis and, according to reference 11, results in a simpler solution. Reference 11 further states that best agreement with experiment is obtained when Görtler's analysis is considered.

Tollmien's solution requires the use of successive approximations. For this reason, and because of the preceding statement, this solution is not reproduced herein. Refer to reference 14 for details of the solution for the two-dimensional and circular jets.

In reference 14, values of U/U_m are tabulated as a function of $y/(x \sqrt[3]{c^2})$, where $\sqrt[3]{c^2}$ is evaluated as 0.063 and c is the scaling factor in Prandtl's mixing length theory. Instead of using $y/(x \sqrt[3]{c^2})$ as the abscissa, it is common practice to plot U/U_m against $y/y_{1/2}$, where $y_{1/2}$ is that value of y for which the velocity is equal to $U_m/2$. Hence, when $U/U_m = 0.5$, $y/y_{1/2} = 1$. For the circular jet, a plot of Tollmien's tabulated data shows $U/U_m = 0.5$ when $y/(x \sqrt[3]{c^2}) = 1.23$; hence, if each abscissa is divided by 1.23, values of $y/y_{1/2}$ are obtained, and the ordinates can then be plotted against these new abscissas. The same procedure was used to replot the slot jet solution. This is the way Tollmien's solutions are plotted in this report.

For the two-dimensional jet, according to Schlichting, Görtler's solution may be written as

$$U = \frac{\sqrt{3}}{2} \sqrt{\frac{K\lambda}{x}} (1 - \tanh^2 \eta_1) \quad (36)$$

$$V = \frac{\sqrt{3}}{4} \sqrt{\frac{K}{\lambda x}} \left[2\eta_1 (1 - \tanh^2 \eta_1) - \tanh \eta_1 \right] \quad (37)$$

where the constant kinematic momentum flux $K = J/\rho$ and

$$\lambda = \frac{1}{2} \sqrt{\frac{U_{m,s}^s}{\epsilon_s}} \quad (38)$$

where s is the fixed distance from the nozzle, $U_{m,s}$ is the centerline velocity at s ,

$$\epsilon_s = \chi_1 b_s U_{m,s}$$

and

$$\eta_1 = \frac{\lambda y}{x}$$

For this case, λ is the empirical constant. Its value was obtained as 7.67 by Reichardt (ref. 22).

For the circular jet, Görtler's solution is

$$U = \frac{3}{8\pi} \frac{K}{\epsilon x} \frac{1}{\left(1 + \frac{1}{4} \eta_2^2\right)^2} \quad (39)$$

$$V = \frac{1}{4} \sqrt{\frac{3}{\pi}} \frac{\sqrt{K}}{x} \frac{\eta_2 \left(1 - \frac{1}{4} \eta_2^2\right)}{\left(1 + \frac{1}{4} \eta_2^2\right)^2} \quad (40)$$

where K is kinematic momentum flux (constant), ϵ is virtual kinematic viscosity (constant), and

$$\eta_2 = \frac{1}{4} \sqrt{\frac{3}{\pi}} \frac{\sqrt{K} y}{\epsilon x}$$

For this case, Reichardt determined the empirical constant \sqrt{K}/ϵ to be 62.1. Equation (39) is identical with equation (19) with ν replaced by ϵ .

Corrsin (ref. 19) reports experimental transverse velocity profiles through region I for a 3-inch (7.62-cm) diameter heated jet and a 1-inch (2.54-cm) diameter heated jet. Distributions were given for values of x/D of 2, 4, and 6 for the 3-inch (7.62-cm) jet and a value of x/D of 5 for the 1-inch (2.54-cm) jet. In region I, the velocity in the potential core remains constant at $U_m = U_0$ and falls off in the transverse direction exterior to the potential core (see fig. 2). In order to compare these experimental velocity distributions with those of Tollmien and Görtler, it was necessary to translate the jet centerline to the border of the potential core at the respective values of x/D , determine the value of $y_{1/2}$ (where $U = U_m/2$) for each of the distributions, and then calculate the abscissa $y/y_{1/2}$ for selected values of U/U_m for each of the distributions considered. Figure 5 shows the comparison of the experimental distributions of reference 19 with the Tollmien and Görtler distributions. The good agreement in figure 5 indicates that the theories of Tollmien and Görtler are satisfactory for use in predicting the velocity profiles in region I of the jet even though these theories were developed for region II.

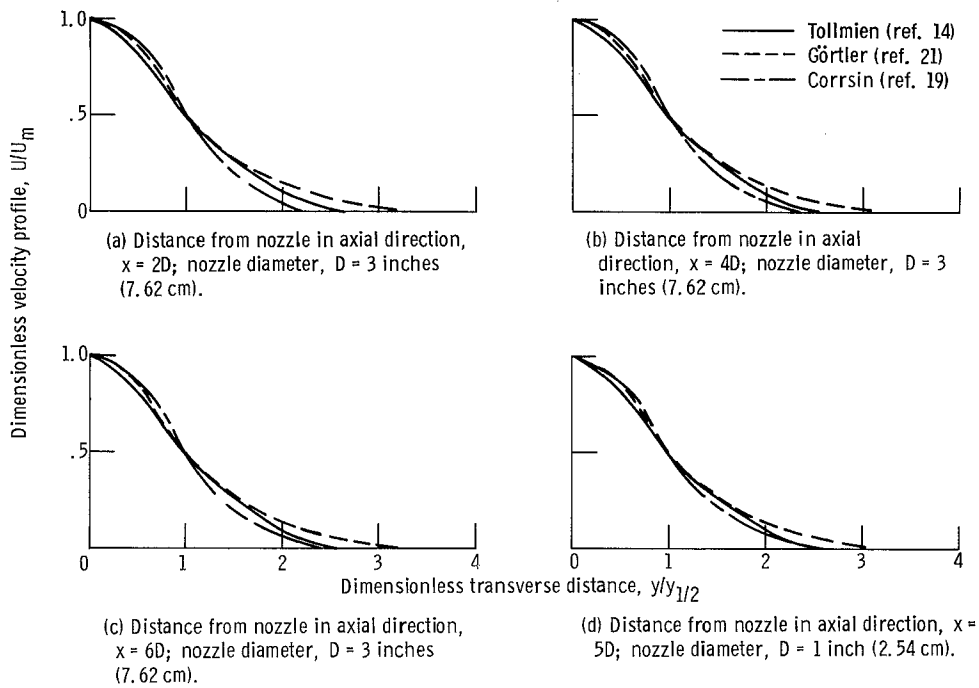


Figure 5. - Velocity profiles in circular turbulent jets.

Under the assumption used in the analyses previously discussed, the transverse velocity distributions differ only by a scale factor. At all sections beyond the tip of the potential core, the velocity distribution is continuous across the line of symmetry, and the ratio σ/x is therefore indicative of the rate of outward spread of the jet. By the same procedure as that explained for the region of flow establishment (region I), reference 13 shows that

$$\frac{U}{U_0} = \sqrt{\frac{1}{\sqrt{\pi}K_1} \frac{B_0}{x}} \exp\left(-\frac{1}{2K_1^2} \frac{y^2}{x^2}\right) \quad (41)$$

for the two-dimensional jet, and

$$\frac{U}{U_0} = \frac{1}{2K_2} \frac{D}{x} \exp\left(-\frac{1}{2K_2^2} \frac{y^2}{x^2}\right) \quad (42)$$

for the circular jet. Equations (41) and (42) show good agreement with experimental data for developed turbulent flow. Equations (41) and (42) were obtained by assuming the same experimental constant σ/x applied to both regions I and II. In addition, they were obtained by use of equation (8), a curve fit of experimental data, rather than from the governing equations considered by Tollmien and Görtler.

Figure 6(a), taken from reference 11, compares experimental data compiled by Förthmann (ref. 23) with the Tollmien and Görtler solutions for a two-dimensional jet and region II. Figure 6(b), also taken from reference 11, compares experimental data compiled by Reichardt (ref. 22) with the Tollmien and Görtler solutions for a circular jet and region II. These curves are plotted with U/U_m as a function of $y/y_{1/2}$; the conversion of Tollmien's solution to this type of plot was discussed previously. The experimental data agree better with Görtler's solution over most of region II, except farther away from the center of the jet, for $y/y_{1/2} > 1.5$.

Reference 1 also presents a plot of experimental data together with the theoretical curves derived by Görtler, namely,

$$\frac{U}{U_m} = \frac{1}{\left[1 + \left(7.48 \frac{y}{x}\right)^2\right]^2} \quad (43)$$

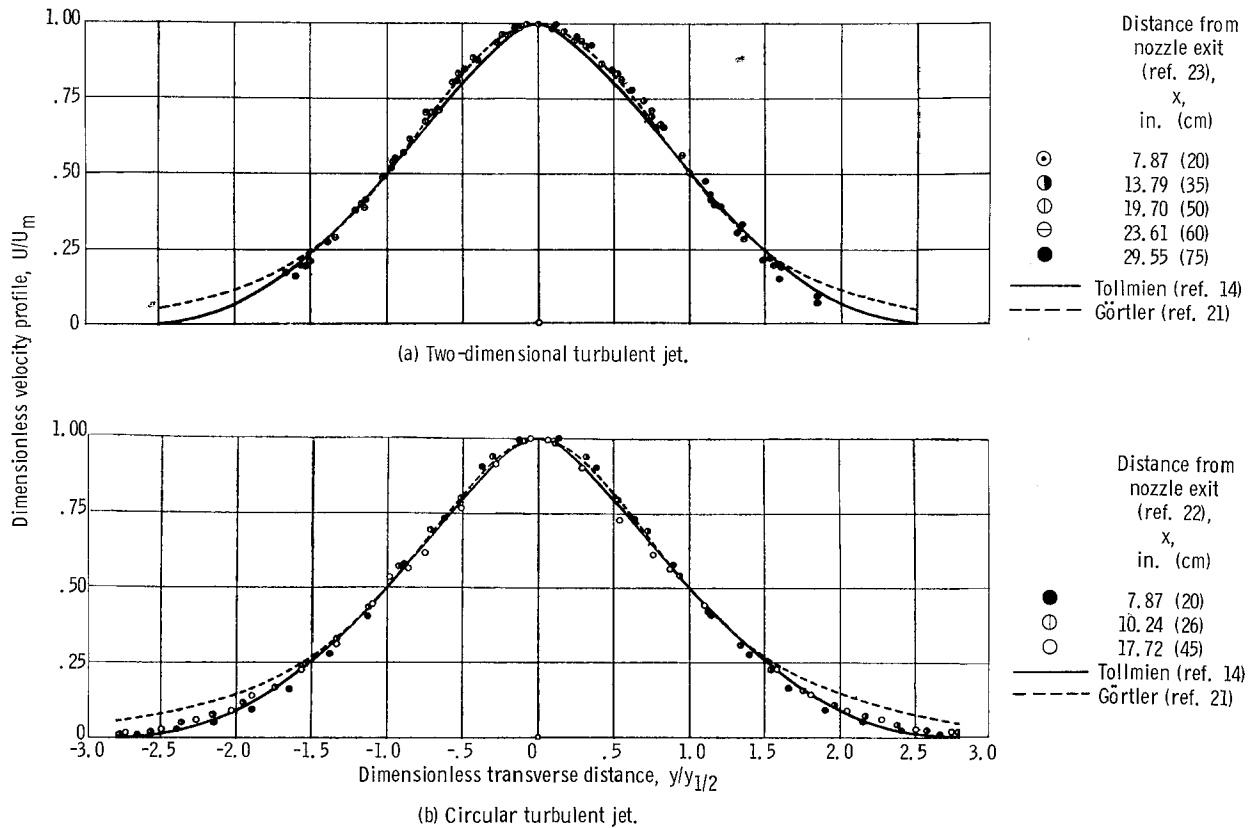


Figure 6. - Velocity distribution in two-dimensional and circular turbulent jets. (From ref. 11).

and an error curve

$$\frac{U}{U_m} = \exp\left(-100 \frac{y^2}{x^2}\right) \quad (44)$$

for the transverse velocity distribution in the region of established flow for a circular jet. The measured transverse velocity distribution is well approximated by either Görtler's solution (eq. (43)) or the error curve (eq. (44)).

In van der Hegge Zijnen (ref. 16), slot jet data are compared against the error curve of equation (44) except that the constant is 70.7 instead of 100. Good agreement is achieved.

Turbulence Distribution Along Centerline

Corrsin (ref. 19) measured the turbulent velocities U'/U_0 for a 1-inch (2.54-cm) diameter heated jet at various distances from the nozzle. A plot of these values along the centerline in region II resulted in a straight line, on log-log paper, with slope equal to -0.88 and intercept at 0.84; that is, the equation of the line may be written as

$$\frac{U'}{U_0} = 0.84 \left(\frac{x}{D}\right)^{-0.88} \quad (45)$$

An experimentally determined value of U'/U_0 at a value of $x/D = 5$ (region I) obtained in reference 19 does not fall on the extension of this line; hence, it appears that in region I the relation of equation (45) does not apply.

Figure 7 shows a normalized turbulence distribution along the centerline for a circular and a slot jet obtained from references 19 and 20, respectively. Reference 19 shows a plot of U_m/U_0 against x/D . By taking the data points for U'/U_0 at any x/D and dividing by the value of U_m/U_0 at the same value of x/D , a curve of U'/U_m against x/D can be obtained. This plot is shown as the solid curve in figure 7. Also

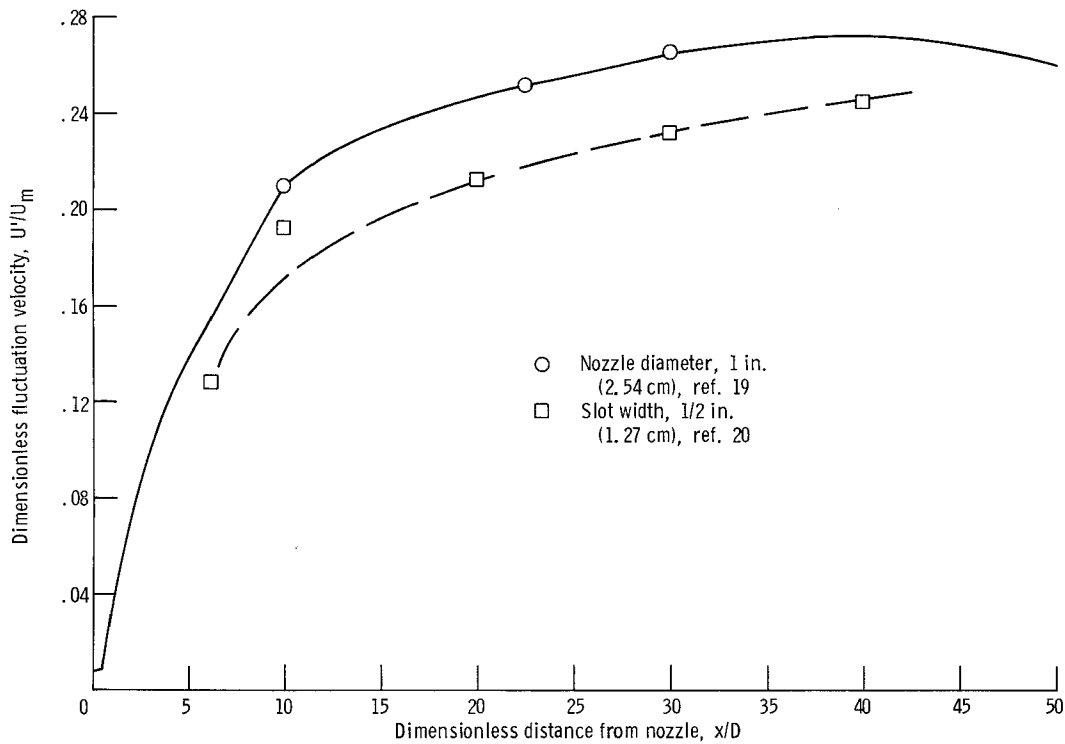


Figure 7. - Turbulence distribution along centerline for a circular and a slot jet.