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MEASUREMENT ERRORS FOR THERMOCOUPLES ATTACHED TO THIN PLATES: APPLICATION TO HEAT FLUX MEASUREMENT DEVICES

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ABSTRACT

Unsteady Surface Element (USE) methods are applied to a model of a thermocouple wire attached to a thin disk. Green's functions are used to develop the integral equations for the wire and the disk. The model can be used to evaluate transient and steady state responses for many types of heat flux measurement devices including thin skin calorimeters and circular foil (Gardon) heat flux gages. The model can accommodate either surface or volumetric heating of the disk. The boundary condition at the outer radius of the disk can be either insulated or constant temperature. Effect on the errors of geometrical and thermal factors can be assessed. Examples are given.

NOMENCLATURE

a – wire radius
A – thermal diffusivity ratio – α_2/α_1
b – disk radius
b^+ – disk radius to wire radius ratio – b/a
B – contact Biot modulus – ha/k_1
Bi – lateral surface Biot modulus – $2h_c a/k_1$
c_m – $\pi(m-1/2)$
G_R – radial Greens function
G_x – x-direction Greens function
h – contact heat transfer coefficient
h_c – lateral heat transfer coefficient
J – Bessel function
k – thermal conductivity
K – thermal conductivity ratio – k_2/k_1
ℓ – wire length
ℓ^+ – ratio of wire length to wire radius – ℓ/a
L – disk thickness
L^+ – ratio of disk thickness to wire radius – L/a
q_0 – heat flux at the disk/wire interface
q^+ – dimensionless heat flux, Eq 6c
q^{++} – dimensionless heat flux, Eq 6e
q_L – heat flux at surface x=L
r – radial coordinate
r' – dummy radial coordinate
s – Laplace transform coordinate
t^+ – non-dimensional time, Eq. 6a
T^+ – non-dimensional temperature, Eq. 6b
T^{++} – non-dimensional temperature, Eq. 6d
x – axial coordinate
x' – dummy axial coordinate

Greek Symbols

α – thermal diffusivity
β_m – roots of $J_0(\beta_m)=0$
γ_m – roots of $J_1(\gamma_m)=0$
τ – dummy time variable

Subscripts

1 – related to the disk
2 – related to the wire
j,i – initial value of a parameter for body j
nl – no heat loss from the wire
nw – value of a parameter if wire is not present
ss – steady state value of a parameter

INTRODUCTION

The operation of a variety of heat flux sensors and calorimeters involves contact temperature measurements on thin plates. Thermocouples are often used for this purpose. Estimating and/or correcting the errors involved in making these measurements is an important problem in experimental heat transfer. Numerous papers have been written on this subject.

For thin skin calorimeters, Burnett (1961) and Larson and Nelson (1969) developed approximate models for estimating the magnitude of the errors. Henning and Parker (1967) and Keltner (1973,1974) developed analytical models for the transient response of intrinsic thermocouples. Keltner and Bickle (1976)

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and Wally (1977) used these response models to correct measurement errors. Cassagne et. al., (1980), Keltner and Beck (1983), and Litkouhi and Beck (1985) developed more accurate transient response models. Wedekind and Beck (1982) addressed the problem of nonuniform heat fluxes. McMurtry and Dolce (1982) developed a numerical model for a fast response calorimeter. Kidd (1985,1986) developed numerical models and used them for sensitivity analyses.

For the circular foil heat flux gages, which are generally called Gardon gages after the developer, Gardon (1953) described the response in terms of a first order or exponential response. Analyses by Ash (1969) and Kirchoff (1972) indicated that the exponential response model was not sufficient for rapid transients. Malone (1967) found that accounting for heat transfer to the center thermocouple wire could significantly affect the shape of the transient response. Keltner and Wildin (1974,1975) developed a response model for the gages and used it to estimate measurement errors. Borell and Diller (1987) analyzed the response to convective heating and developed convective calibration methods.

The errors involved in making temperature measurements with thermocouples attached to thin plates may be transient, steady state, or both. The errors may result from the thermocouple installation altering the local surface temperature distribution or the effects of heat transfer in the thermocouple/plate combination. This paper will deal with the latter problem. There are many sources of this type of error, but the most significant are:

1. thermal constriction effects within the plate to which the thermocouple is attached,
2. thermal inertia of the thermocouple,
3. imperfect contact between the thermocouple and the surface,
4. heat loss from the thermocouple to the ambient,
5. the effective junction location being displaced from the surface.

Keltner and Beck (1983) developed the Unsteady Surface Element (USE) methods that are applied to a model of a thin disk attached to a wire. In this paper, Green's functions are used to develop the integral equations describing the temperature of the wire and the disk. The model can accommodate either surface or volumetric heating of the disk. The boundary condition at the outer radius of the disk can be either insulated or constant temperature. The model can be used to evaluate transient and steady state responses for many types of heat flux measurement devices including thin skin calorimeters and circular foil heat flux gages. The effect on the errors of geometrical factors, such as the disk to wire radius ratio or the ratio of disk thickness to wire radius, and thermal factors, such as contact resistance between the wire and the disk or heat loss from the wire, can be assessed.

A sketch of the model is shown in Figure 1. The disk portion of the model is treated in a two-dimensional fashion. The thermocouple wire is modeled as one-dimensional; heat conduction occurs only in the axial direction wire. A fin correction can be used to allow for heat loss from the thermocouple. Imperfect thermal contact at the interface of the disk and the wire is modeled by a contact heat transfer conductance, h.

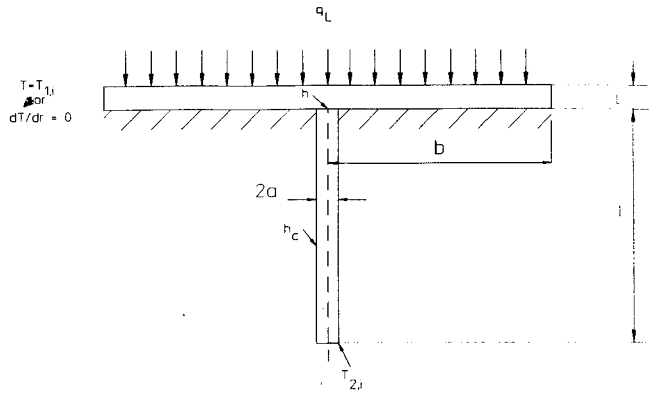

Figure 1. Model Geometry

The response models are developed for a step change in either the initial temperature or the surface heat flux. For surface heating, the initial temperature of the disk is the same as that of the wire. For volumetric heating, the initial temperature of the disk is different from that of the wire and the surface heat flux is zero. The response to a time varying condition of either type can be obtained from the step response via convolution.

MATHEMATICAL FORMATION

The heat transfer at the interface of the wire and the disk can be expressed:

$$q_{0,1} = h(T_2(t) - T_1(t)) \tag{1}$$

where h is the contact heat transfer conductance. For perfect contact, h is infinite, resulting in $T_2(t) = T_1(t)$.

By energy conservation, the area averaged heat flux entering body 1 at the interface is equal to that leaving body 2, or:

$$q_{0,1} = -q_{0,2} \tag{2}$$

The temperature at x=0 for the disk is given by (Beck, et. al. 1988):

$$\begin{aligned} T_1(r,0,t) = {} & 2\pi\alpha_1\int_{\tau=0}^{t}\int_{r'=0}^{b} (q_L/k_1) * G_{ROJ}(r,t|r',\tau)G_{X22}(0,t|L,\tau)r'dr'd\tau \\ & +2\pi\alpha_1\int_{\tau=0}^{t}\int_{r'=0}^{a}(q_{0,1}(\tau)/k_1) * G_{ROJ}(r,t|r',\tau)G_{X22}(0,t|0,\tau)r'dr'd\tau \\ & +2\pi\int_{x'=0}^{L}\int_{r'=0}^{b}T_{1,i}G_{ROJ}(r,t|r',0) * G_{X22}(x,t|x',0)r'dr'dx' \end{aligned} \tag{3}$$

In equation 3, the first term on the right hand side represents the effect of surface heating, the second term the effect of heat loss to the thermocouple wire, and the third term the effect of the initial temperature in the disk. G_X represents the x-direction Green's function; whereas G_R

represents the radial direction Green's function. The numbering system utilized for the Green's function is that developed by Beck and Litkouhi (1988). The numeral subscripts indicate the boundary conditions: J=0 is an infinite boundary, J=1 indicates a prescribed temperature boundary condition, and J=2 indicates a prescribed heat flux boundary condition.

In this paper, two cases are considered. If there is volumetric heating of the disk, then $q_L = 0$; this case is modeled by a step change in the initial temperature of the disk. If there is surface heating of the disk, then $T_{1,i} = 0$. Without loss of generality, $T_{2,i}$ can be set equal to zero. For the insulated boundary case, the third term is equal to the initial temperature of body 1 ($T_{1,i}$).

The average temperature over the disk/wire interface is the concern of this paper. The average temperature over the area $0 \le r \le a$ can be expressed as:

$$\bar{T}_1(t) = \frac{1}{\pi a^2} \int_{r=0}^{a} T_1(t) 2\pi r \, dr \tag{4}$$

The average non-dimensionalized temperature for the case of impulsive, volumetric heating is given by:

$$\begin{aligned} \bar{T}_1^+(t_a^+) = & - T_{1,i}/(T_{1,i} - T_{2,i}) \\ & + \frac{4\pi}{a} \int_{\tau_a^+=0}^{t_a^+} \int_{r=0}^{a} \int_{r'=0}^{a} q_0^+(\tau_a^+) G_{R0J}(r, t_a^+ | r', \tau_a^+) * \\ & \qquad G_{X22}(0, t_a^+ | 0, \tau_a^+) r' r dr' dr d\tau_a^+ \qquad (5a) \\ & + \frac{4\pi}{a^2} \int_{x=0}^{L} \int_{r=0}^{a} \int_{r'=0}^{b} G_{R0J}(r, t_a^+ | r', 0) * \\ & \qquad G_{X22}(x, t_a^+ | 0, 0) r' r dr' dr dx' \end{aligned}$$

Whereas that for surface heating can be expressed as:

$$\begin{aligned} \bar{T}_1^{++}(t_a^+) = & \frac{4\pi}{a} \int_{\tau_a^+=0}^{t_a^+} \int_{r=0}^{a} \int_{r'=0}^{b} G_{R0J}(r, t_a^+ | r, \tau_a^+) * \qquad (5b) \\ & \qquad G_{X22}(0, t_a^+ | L, \tau_a^+) r' r dr' dr d\tau_a^+ \\ & + \frac{4\pi}{a} \int_{\tau_a^+=0}^{t_a^+} \int_{r=0}^{a} \int_{r'=0}^{a} q_o^{++}(\tau_a^+) G_{R0J}(r, t_a^+ | r', \tau_a^+) * \\ & \qquad G_{X22}(0, t_a^+ | 0, \tau_a^+) r' r dr' dr d\tau^+ \end{aligned}$$

where $$t_a^+ = \alpha_1 t/a^2 \tag{6a}$$

$$T_j^+ = (T_j - T_{j,i})/(T_{1,i} - T_{2,i}) \tag{6b}$$

$$q_0^+ = q_{0,1} a/\{k_1 (T_{1,i} - T_{2,i})\} \tag{6c}$$

$$T_j^{++} = T_j/(q_L a/k_1) \tag{6d}$$

$$q_0^{++} = q_{0,1}/q_L \tag{6e}$$

The wire is considered to have conduction in the axial direction only. The nondimensional temperature of the wire at x=0 can be expressed (Beck, et. al. 1988):

$$T_2^+(t_a^+) = -a \frac{A}{K} \int_{\tau_a^+=0}^{t_a^+} q_0^+(\tau_a^+) G_{X21}(0, t_a^+ | 0, \tau_a^+) d\tau_a^+ \tag{7a}$$

or

$$T_2^{++}(t_a^+) = -a \frac{A}{K} \int_{\tau_a^+=0}^{t_a^+} q_0^{++}(\tau_a^+) G_{X21}(0, t_a^+ | 0, \tau_a^+) d\tau_a^+ \tag{7b}$$

where: $K = k_2/k_1$ and $A = \alpha_2/\alpha_1$ (8)

The fin approximation is used to allow for heat loss from the wire (Beck, et. al., 1988).

$$T_2(t_a^+) = T_{2,nl} \exp(-BiAt_a^+) \tag{9}$$

where: $Bi = 2ha/k_2$ (10)

Many of the Greens functions are in the form of infinite series; as time approaches zero, a very large number of terms are necessary for accurate evaluation. The time partitioning method outlined in Keltner and Beck (1987) allows the use of simpler expressions. Expressions for the different Green's functions in the necessary time partitions are given in the Appendix. The Greens functions are from Beck, et. al., 1988.

Using equation 6, it can be shown that non-dimensional forms of equation 1 are:

$$q_0^+ = B\{T_2^+ - T_1^+\} \tag{11a}$$

$$q_0^{++} = B\{T_2^{++} - T_1^{++}\} \tag{11b}$$

The Laplace transforms of equations (5), (7), and (11) are taken; Equations (5) and (7) are substituted into Equation (11). The resulting equation can be solved for heat flux at the interface. From this solution and Equation (7), $T_2^+(t_a^+)$ or $T_2^{++}(t_a^+)$ can be determined. The Gaver-Stehfest method of numerical inversion is used to evaluate the equations. (Stehfest, 1970)

A Fortran model was developed using this formulation. Variables effecting the behavior of the response of the thermocouple are geometric parameters (b^+, L^+, and ℓ^+), thermophysical property ratios (K and A) and heat transfer characteristics (Bi, and 1/B). The effect of varying these parameters can be examined with the model.

Case 1 - Volumetrically heated, insulated boundary gage

X-Ray dosimeters undergo impulsive, volumetric heating. Assuming an insulated boundary at r=b and a very long wire provides a good model for this type of calorimeter.

The response of these devices can be modeled by assuming that the disk and the wire start at different temperatures, i.e., $T_{1,i} \neq T_{2,i}$. The error in this case is defined as the difference between $T_2(t)$ and $T_{1,i}$ because an ideal sensor would instantaneously indicate the initial disk temperature.

By using very large values of b^+ and L^+, this model can also be used for a wire attached to a semi-infinite body which undergoes a step change in temperature. The semi-infinite response for the present model is compared with the response for the same conditions (K=A=1, 1/B=Bi=0) from Keltner and Beck (1983) in Figure 2. The maximum difference between the responses from the two models is 3% which occurs at $t_a^+=0.5$. The difference is due to the use of more accurate expressions for the Green's functions in the present analysis.

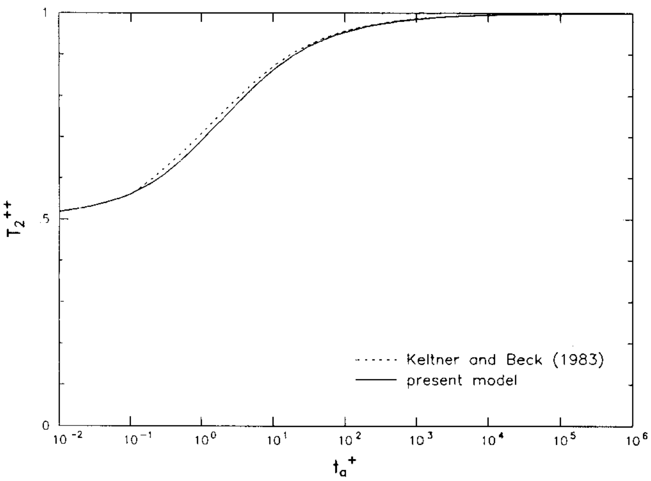

Figure 2. Case I - Comparison of the current model with an earlier model for the response of an intrinsic thermocouple on a thick wall. (K=A=1 and 1/B=Bi=0)

Figure 3 shows the effect of the ratio of the disk thickness to the wire radius on the response of the thermocouple. These responses are for similar metals (K=A=1) with no heat loss from the wire and perfect contact at the interface. The response for L^+ values ranging from 0.2 to 5 are compared to the response for an ideal intrinsic thermocouple attached to a semi-infinite body. The boundary at x=L begins to affect the response of the wire at $t_a^+=0.1L^{+2}$; however, the response does not vary significantly from the semi-infinite response until approximately an order of magnitude longer. Except for the early times, thin disks respond more slowly than the thicker ones. For large disk-to-wire radius ratios and L^+ values greater than 10, the response approaches the case of an ideal intrinsic thermocouple attached to a semi-infinite body; the maximum difference between the response for $L^+=10$ and the semi-infinite response is 0.15%.

Eventually, energy conducted from the disk into the wire will affect the response. This effect is dependent upon the combination of L^+ and b^+. One method of examining this effect is to hold L^+ constant and vary b^+. For $L^+=2$ and b^+ values ranging from 20 to 1000, the heat loss from the disk begins to have an effect at approximately $t_a^+=(b^+-1)^2$. Figure 4 shows the response for disks with these geometric parameters, with K=A=1, no contact resistance or heat loss from the wire. The temperature of the disk would become equal to that of the wire at very late times.

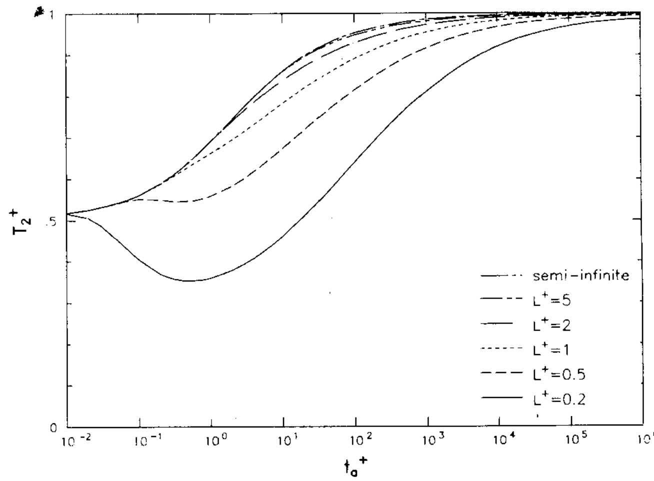

Figure 3. Case I - The effect of disk thickness on the response of an intrinsic thermocouple ($b^+=1000$, K=A=1, and 1/B=Bi=0).

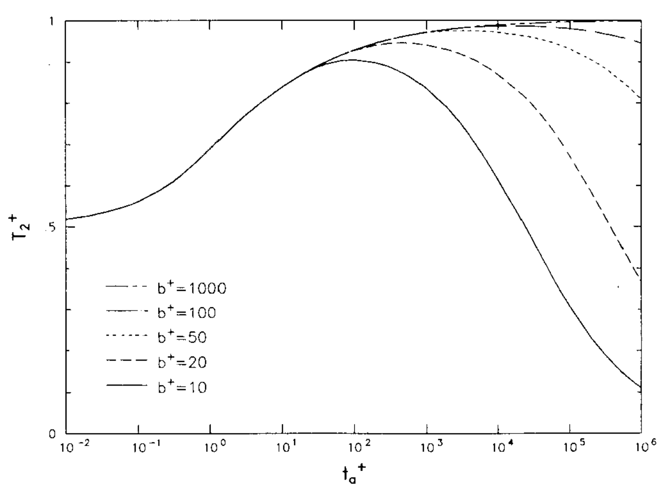

Figure 4. Case I - The effect of the disk radius/wire radius ratio on the response of an intrinsic thermocouple ($L^+=2$, K=A=1, and 1/B=Bi=0).

Material property effects are shown in Figure 5 for an ideal intrinsic thermocouple (1/B=0) with no heat loss from the wire (Bi=0) attached to a disk with the following geometric properties: $L^+=2$, $b^+=1000$, ℓ^+ approaching infinity. The response is much slower for larger values of $K/\sqrt{A}$. The very long time response is unity for all values of $K/\sqrt{A}$, however

Heat lost from the thermocouple will also drive the response to zero. The effect of varying rates of heat loss (values of Bi) from the wire for a gage with $L^+=2$, $b^+=100$, no contact resistance (1/B=0), and made from similar materials (K=A=1) is shown in Figure 6. At early times, the heat loss has little effect. As the wire heats, this loss becomes more important and the response falls below the zero loss case.

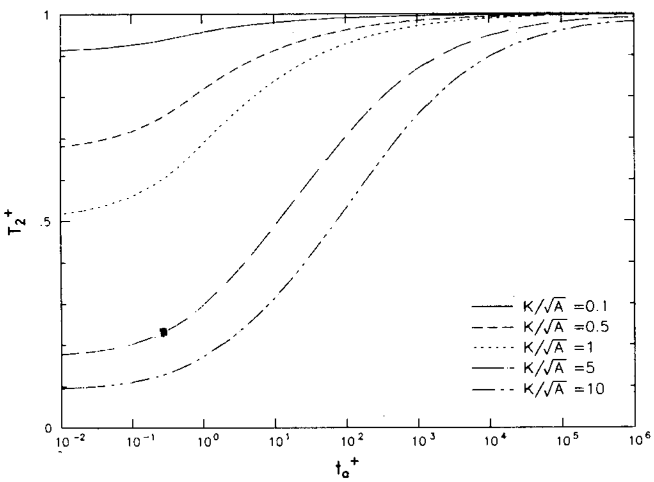

Figure 5. Case I - The effect of thermal properties on the response of an intrinsic thermocouple ($L^+=2$, $b^+=1000$, and $1/B=Bi=0$).

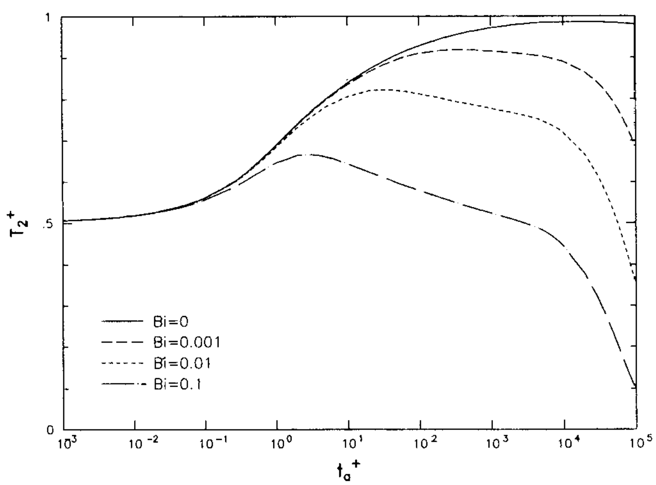

Figure 6. Case I - The effect of heat loss from the wire on the response of an intrinsic thermocouple ($L^+=2$, $B^+=100$, $K=A=1$, and $1/B=0$).

The effects of using beaded thermocouples or having contact resistance at the disk/thermocouple interface were discussed in Keltner and Beck (1983). In both cases, the zero time response is zero and the temperature rise is slower. The effects of contact resistance are incorporated in the model through the parameter B. The effect of a beaded thermocouple in displacing the junction from the interface can be obtained from $T_2(t)$ via convolution.

Case 2: Surface Heated, Insulated Boundary Gage

Thin skin calorimeters and certain types of laser power meters are examples of instruments that are represented by a model with a surface heat flux on the front surface of the disk ($x=L$) and an insulated radial boundary (at $r=b$). Such calorimeters are frequently used in wind tunnel testing. They have the advantage of being easy and inexpensive to construct. The ideal response of such is a gage is a linear increase of temperature following a short transient.

A design that was previously analyzed by Keltner and Bickle (1976) involved a thin skin calorimeter with a 36 gage (.127 mm) type K thermocouple (chromel/alumel) intrinsically attached to a 1 mm thick 304 stainless steel plate. The wire is very long compared to its diameter. The resulting value of L^+ is 15.7 with b^+ and ℓ^+ very large. For the chromel wire, $K=1.13$ and $A=1.27$; whereas for the alumel wire $K=1.75$ and $A=1.88$. The gage is considered to have no interfacial resistance to heat flux ($1/B=0$) or heat loss from the wire ($Bi=0$).

The resulting responses are shown in Figure 7. Also given is the ideal temperature or the average non-dimensional temperature for the substrate over the contact area, $0\le r\le a$, if no wire was present. At "late" times, the ideal response is a ramp of the form $C{*}t$. A value of t_a^+ of 1000 represents a real time of approximately 1 second for this gage design.

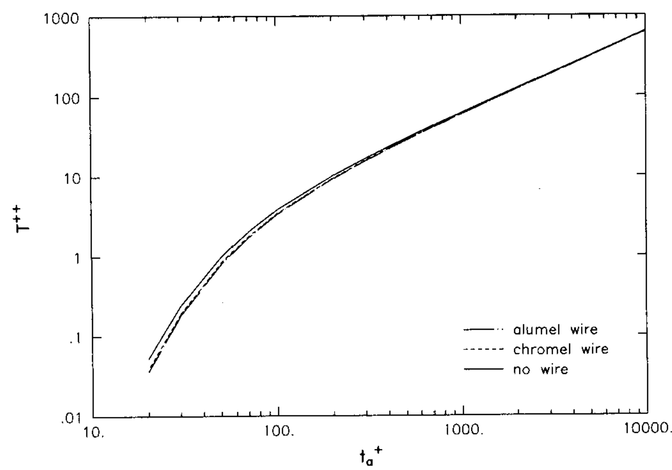

Figure 7. Case 2 - Ideal and predicted responses of the chromel and alumel junctions of an intrinsic thermocouple on a stainless steel thin skin calorimeter. ($L^+=15.75$, $b^+=\ell=10000$, and $1/B=Bi=0$)

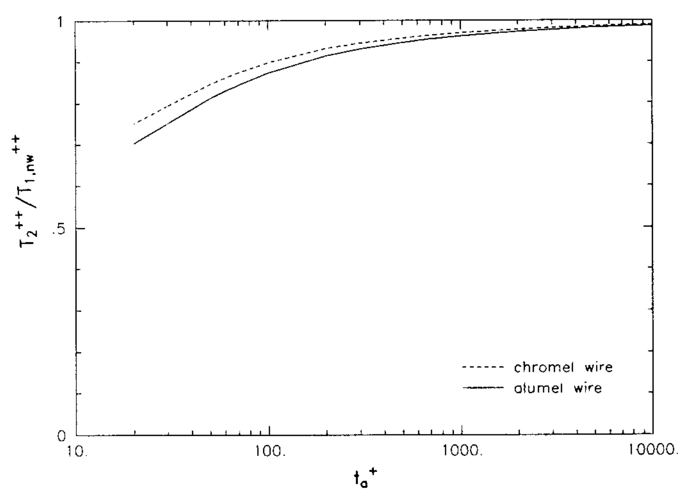

Figure 8. Case 2 - Ratio of the actual respones to the ideal response of a thin skin calorimeter ($L^+=15.75$, $b^+=\ell^+=10000$, and $1/B=Bi=0$).

The ratio of the actual response to the ideal response is considered to be the difference between the error and unity. In general, the item of interest is the slope of the ramp because this is directly related to the heat flux. It can be shown that the time dependent error in the slope of the ramp is equal to C(1 - step response), where the step response was defined in case 1. This value is given in Figure 8. At t_a^+=1000, the error is 3% for the chromel wire and 4% for the alumel wire.

Case 3: Surface Heated Constant Temperature Boundary Gage

A circular foil (Gardon) heat flux gage can be represented by a gage which experiences surface heating and has a constant temperature at the radial boundary (r=b). Such gages often consist of a copper wire attached to a constantan disk (K=16.1, A=17.0). For the gage analyzed by Keltner and Wildin (1974, 1975), the geometric parameters are L^+=1.875, b^+=45, and ℓ^+=90.6 for a wire radius of 0.0016 in. The response for such a gage is compared to the ideal response, that is the temperature if no wire was attached, in Figure 9. The gage achieves a steady state response at t_a^+=3000 which corresponds to an real time of 0.75 seconds. For a 30 W/cm^2 flux, the steady state value of 215 represents a 120 °C temperature difference between the center of the disk and its edge. The ratio of the response to the ideal response is shown in Figure 10. The ratio of the steady state response to the ideal steady state response is 0.794.

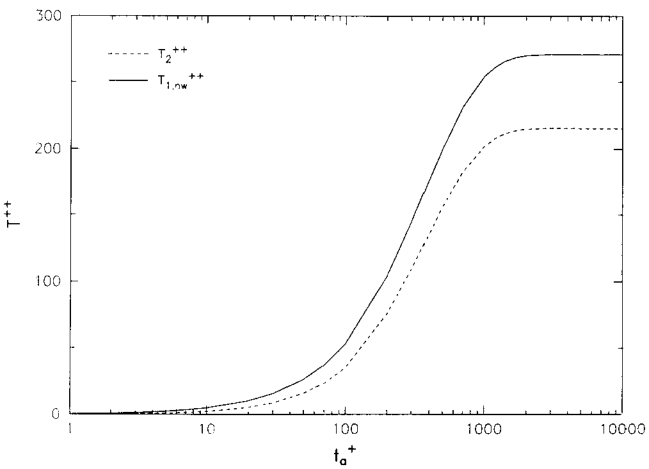

Figure 9. Case 3 - The response of a Gardon Gage to a constant flux (L^+=1.875, b^+=45, ℓ^+=90.6, K=16.1, A=12, 1/B=Bi=0)

Keltner and Wildin (1974) analyzed a gage with the same parameters. The normalized responses (the response divided by the steady state response) are compared in Figure 11. Although the normalized responses are similar, the present model predicts a ratio of the steady state response to the ideal steady state response of 0.794 compared to a value of 0.830 for Keltner and Wildin (1974). The difference in steady state values of 4.3% may be due to the fact that Keltner and Wildin (1974) used the centerline temperature (r=0) instead of the average over the interfacial area (0≤r≤a).

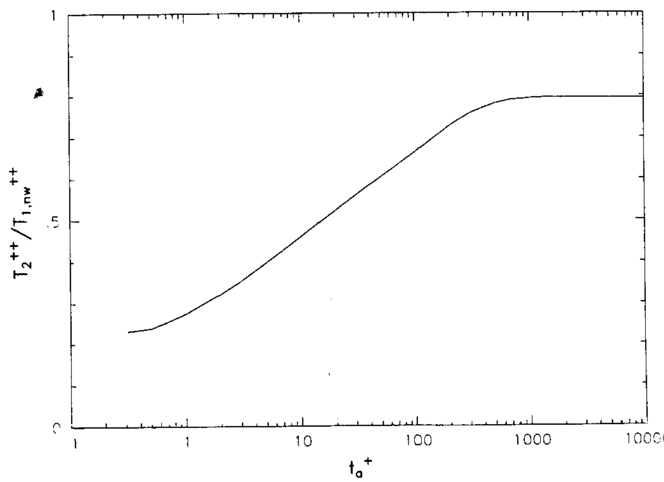

Figure 10. Case 3 - Ratio of the actual response to the ideal response for a Gardon Gage.

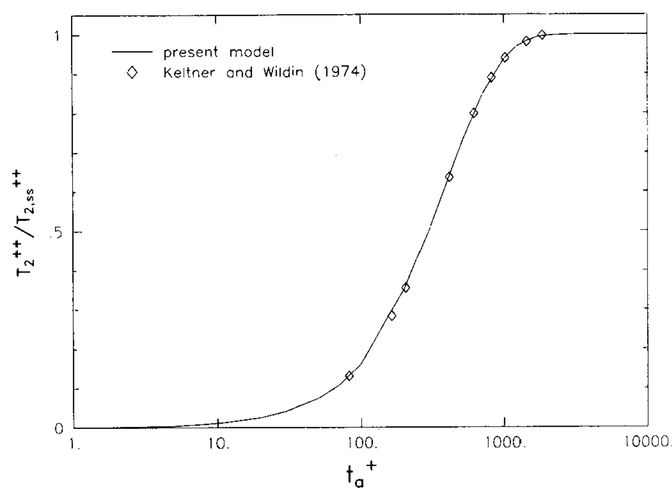

Figure 11. Comparison of the response of a Gardon Gage predicted from the current work and an earlier work.

SUMMARY

Using the unsteady surface element method and Green's function integral equations, a model of a thermocouple attached to a thin disk has been developed. The model can be adapted to a variety of heat flux gages by varying flux, initial, and boundary conditions. Varying a few geometric, thermophysical, or heat transfer properties allows the model to be applied to many different situations.

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The computer program written for this analysis can be released after approval of a written request submitted to the lead author.

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