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A Vortex Model for Wall Flame Height

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16. Abstract

A two-dimensional vortex model is developed to describe flames on burning walls. The flame is considered a region of intense vorticity generation and is modeled by an equivalent vortex filament. Flame height is predicted by matching the induced air-flow to stoichiometric requirements based on wall mass loss rate or energy release rate. The vortex model predicts the same two-thirds power law relationship that has been determined from other approaches. The quantitative predicted height is within the published limits of experimental certainty.

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This is because shrinking δ to zero removes the line integrals in the x - direction, and outside the flame the y velocity vector is approximated as zero. Thus, Bjerknes theorem leads to the relationship that the velocity of gases in the flame are proportional to the square root of the distance from the base of the flame. Since the circulation around a loop is identical to the total vorticity of the area bounded by the loop, equation 5 shows that the vorticity increases higher into the flame.

At this point, it is convenient to amass this dispersed vorticity into a single vortex filament. The vertical location of the vortex will be taken at the point where half the vorticity of the flame is above it. Since the circulation integral goes like the integral of the square root of y, the location of the vortex, y_v will be taken when

$$y_v^{3/2} = \frac{(y_f)^{3/2}}{2} \qquad (6)$$

or when y_f is 1.59 y_v. Going back to equation 5 and integrating the velocity through the entire flame, the total vortex strengh at y_v can be taken as

$$\Gamma = \frac{2}{3}\sqrt{\frac{(T_F - T_A)}{T_A}\, g}\; y_f^{3/2} \qquad (7)$$

This vortex will be located at a close but unspecified distance, δ_v, from the burning wall.

At this point, the model of the vortex and its strength does not provide any relationship to the burning rate of the wall. To develop this relation, two further relationships are needed. First, it will be assumed that the flame tip occurs when enough oxygen has been brought into the flame to combust the burning wall volatiles at a stoichiometric ratio. Second, the vortex strength must be used to find the air induced into the flame. The first of these can be stated as:

$$\dot{m}_A = \left(\frac{a}{f}\right)_s \dot{m}_f \qquad (8)$$

The second requires a measure of the total flow induced by the vortex.

Figure 3 shows the type magnetic field produced between two buss bars. The magnetic field lines are the analog to the velocity streamlines in vortex theory. From the illustration in this case, all inflow ends at the midsection of the buss bar. From there, outflow begins. In this vein, the gas flow between the vortex location and the wall will be determined as shown in figure 4. That is,

$$\dot{m}_A = \rho_F V_v \delta_v \qquad (9)$$

Figure 4 also shows a mirror vortex on the other side of the wall. This vortex is used to create a flow field preventing velocity components normal to the wall. To find the flow field induced by these vortices of opposite sense, the Biot Sarvart law is used as described in reference 6.

$$\vec{V} = \frac{\Gamma}{4\pi}\int \frac{\vec{dl} \times \vec{r}}{r^3} \qquad (10)$$

The induced velocity from each filament at the wall will consequently be

$$V_I = \frac{\Gamma}{2\pi\ \delta_v} \qquad (11)$$

If this is taken as the velocity induced in the flame at this point, then the volumetric flow will be given as

$$V_I \, \delta_v = \frac{\Gamma}{\pi} \qquad (12)$$

Employing equation 7 and 12 and multiplying by flame density,

$$\rho_F \, V_I \, \delta_v = \rho F \frac{2}{3\pi} \sqrt{\frac{(T_F - T_A)\, g}{T_A}} \; y_F^{3/2} \qquad (13)$$

Using equation 8, equation 13 becomes

$$\left(\frac{a}{f}\right)_s \dot{m}_f = \rho F \frac{2}{3\pi} \sqrt{\frac{(T_F - T_A)\, g}{T_A}} \; y_F^{3/2} \qquad (14)$$

Equation 14 shows that the flame height is proportional to the burning rate (or energy release rate) to the two-thirds power as shown in equation 1.

DISCUSSION

The form taken by the results of a vorticity analysis differs from the forms yielded in other analyses, primarily from the fact that the specific heat and the entrainment coefficient do not appear at all in the vorticity analysis. For comparison with equation 1, which has the parameters used for data analysis in this area, some reworking of equation 14 is necessary.

Rearranging terms results in

$$y_f = \left[\left(\frac{a}{f}\right)_s \frac{\dot{m}_f}{\rho_F} \frac{3\pi}{2} \sqrt{\frac{T_A}{(T_F - T_A)\, g}}\right]^{2/3} \qquad (15)$$

Assuming that the mass loss rate, $\dot{m}_f$, can be converted to energy release rate, $\dot{E}$, by multiplication by some heat of combustion, ΔHc,

$$\dot{E} = \dot{m}_f \; \Delta Hc \qquad (16)$$

then equation 15 can be written as

$$y_f = \left[\left(\frac{a}{f}\right)_s \frac{\dot{E}}{\rho_A \Delta Hc} \frac{3\pi}{2} \sqrt{\frac{T_F^2}{T_A\,(T_F - T_A)\, g}}\right]^{2/3} \qquad (17)$$

This can be converted to a form like equation 1 by multiplying numerator and denominator by the missing terms to get

$$y_f = \left[\left(\frac{a}{f}\right)_s \frac{3\pi}{2\,\Delta Hc} \sqrt{\frac{T_F^2}{T_A\,(T_F - T_A)}} \left(C_p \; T_A\right)\right]^{2/3} \left[\frac{\dot{E}}{C_p \; T_A \; \rho_A \; \sqrt{g}}\right]^{2/3} \qquad (18)$$

To compare equation 18 with equation 1, the first term on the right hand side has to be evaluated. The parameters used are as follows:

$$\left(\frac{a}{f}\right)_s = 10$$

$$\Delta Hc = 10 \text{ k J/g}$$

$$C_p = .24 \text{ cal/g } ^\circ\text{C}$$

$$T_F = 1800^\circ \text{ F}$$

$$T_A = 70^\circ \text{ F}$$

Using these numbers,

$$y_f = 5.0 \left[\frac{\dot{E}}{C_p \; T_A \; \rho_A \; \sqrt{g}} \right]^{2/3} \tag{19}$$

Given the uncertainties in flame height measurements as well as the different assumptions used in a vortex approach, the close agreement between equation 19 and equation 1 is remarkable. In experiments by Hasemi quoted in reference 3, the coefficient would be 2.9 for the height of continuous flame and 6.1 for the upper most flame tips.

The common finding between this vorticity approach and previous studies is that $\dot{E}/\sqrt{g}$ to the two-thirds power or $\dot{m}/\sqrt{g}$ to the two-thirds power is the most important pair of terms for flame height correlations. However, the development of the vortex model raises questions about the importance of other factors in accurately modeling flame height. The vortex model induces only enough air to combust the pyrolyzing material while the entrainment coefficient used in reference 3 would involve entraining between 3 and 6.6 times stoichiometric air into the flame. This issue could be resolved by experiment. The entrainment theories incorporate the specific heat as a primary parameter while the vortex theory circumvents this requirement. The reason C_p appears in the right-hand side of equation 19 is that its inverse is included in the factor 5.

The vorticity model clearly demonstrates the wall flame as a region of intense vorticity generation. Although this vorticity was rolled into a single filament for calculational purposes, the actual flame vorticity is of necessity dispersed and disorganized. In actual flames, the vorticity cannot roll into a row of discrete vortices because the wall condition would place mirror vortices directly opposite to them. It is known that the only stable situation involves staggered vortices. This vortex instability can have some influence on wall flame tip oscillation.

While the vortex model does conveniently explain wall flame height and provide insight into flame stability, the major weakness of the model is that it does not explicitly give a flame thickness. Thus, without further boundary layer type analyses, the vortex model's utility is limited insofar as determining heat transfer to the wall for flame spread rate prediction.

CONCLUSIONS

Development of a vortex model to predict wall flame height leads to the following conclusions:

1. The wall flame height is proportional to either the mass injection rate or the energy release rate to the two-thirds power as predicted by other theories and found in experiment.

2. The coefficient to the standard heat release rate array of terms predicted by the vortex model is well within experimental uncertainty.

3. An imaginary vortex filament can satisfactorily represent the dispersed vorticity of the flame for calculations.

4. The velocity increases with square root of height as the flame is traversed vertically.

5. The vorticity of the flame causes air for combustion to be added by an induction mechanism.

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EXECUTIVE SUMMARY

Vorticity theory has been successfully used to theoretically predict and explain lift forces on wings, ground effects during aircraft takeoff and landing, and minimum separation standards for avoiding trailing vortices from large aircraft. Although aircraft wings are relatively large areas of vorticity generation, in many applications these regions can be treated as vortex filaments. While aircraft vorticity generation is ultimately caused by frictional effects, vorticity in fires is often caused by pressure and buoyancy forces. If a fire plume is viewed as a region of vorticity production somewhat analogous to a lifting surface, the possibility of approximating flow effects by a vortex filament arises as a reasonable approach.

In this analysis, two classical vortex equations (Bjerknes theorem and the Biot-Savart law) are used to characterize the vorticity produced by a burning wall. The vorticity, which is dispersed throughout the flame, is idealized by a single vortex filament located in the upper half of the visible flame. By employing a mirror vortex element behind the wall, the air induced into the flame can be calculated. This influx of air, coupled with the stoichiometric requirements for complete combustion of the fuel evolved from the wall, leads to an analytic relationship for the size of the flame based on either the mass loss rate of the wall or the energy release rate.

The final relationship agrees well with theoretical and experimental results recently published. The use of vortices to describe flow fields caused by fires offers a powerful method to analyze specific fire problems that might be extremely difficult to attack by other techniques.

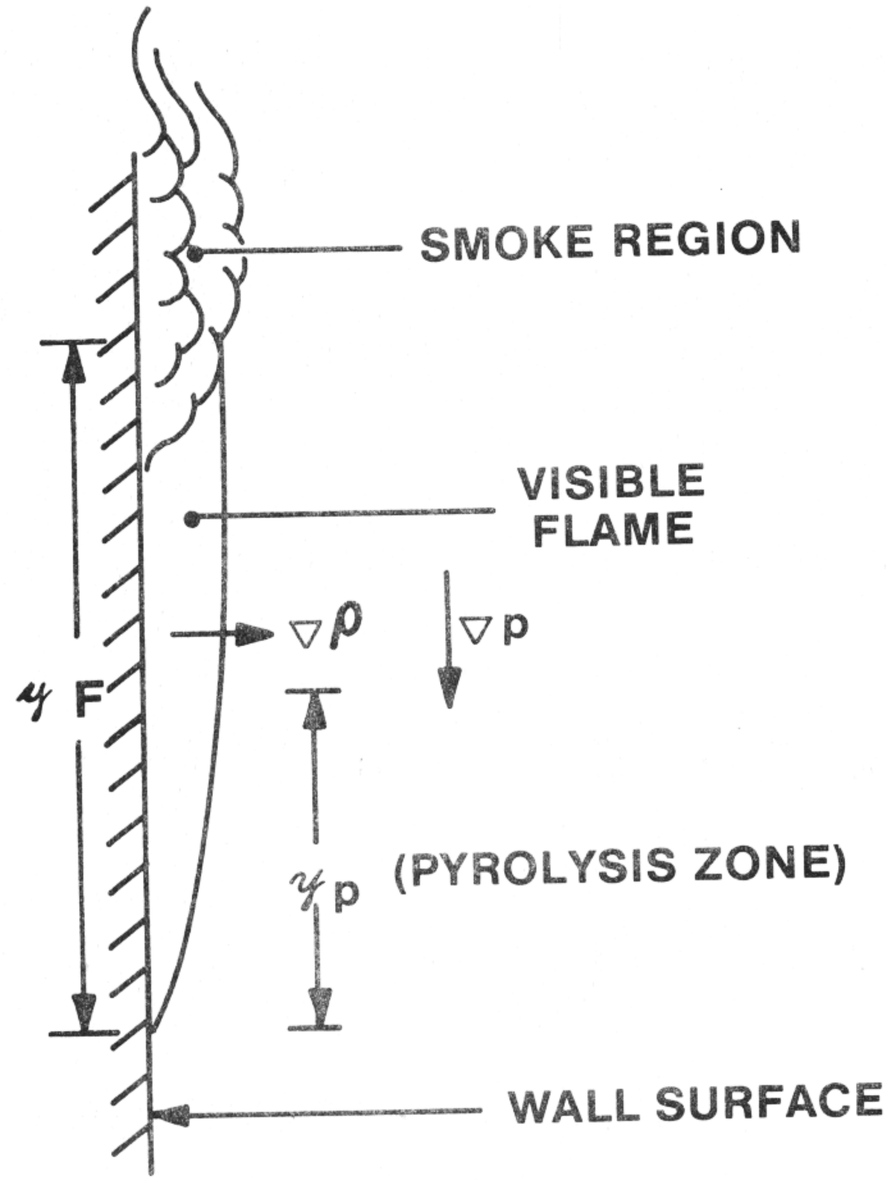

FIGURE 1. IDEALIZED WALL FLAME

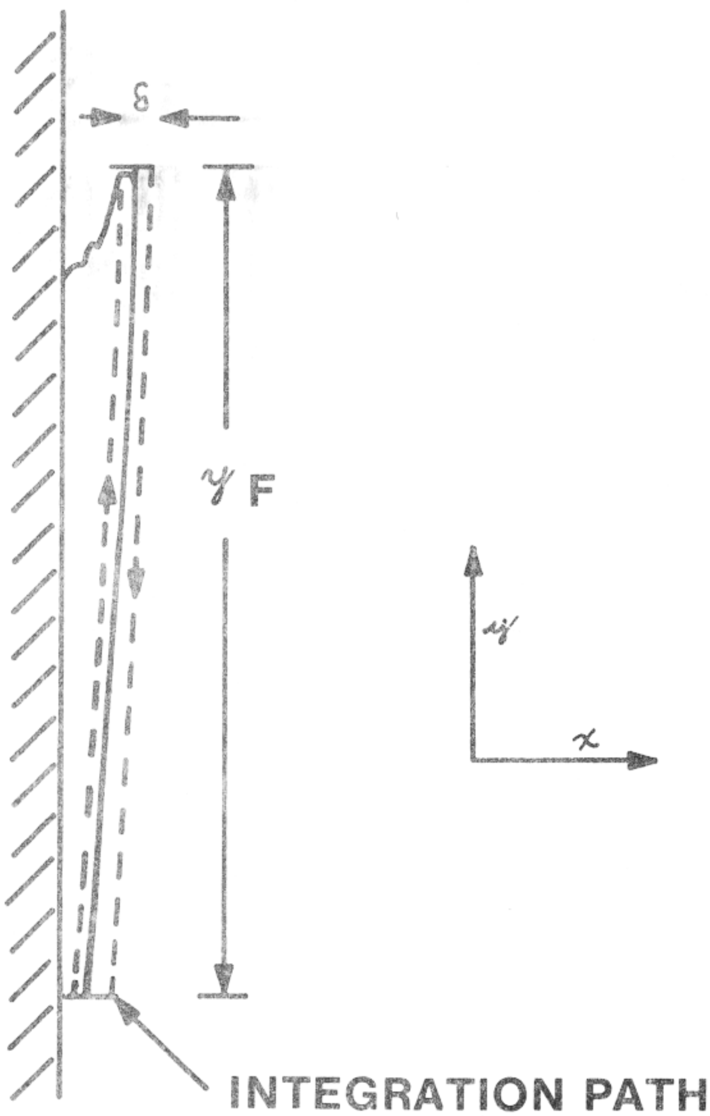

FIGURE 2. CIRCULATION INTEGRATION PATH

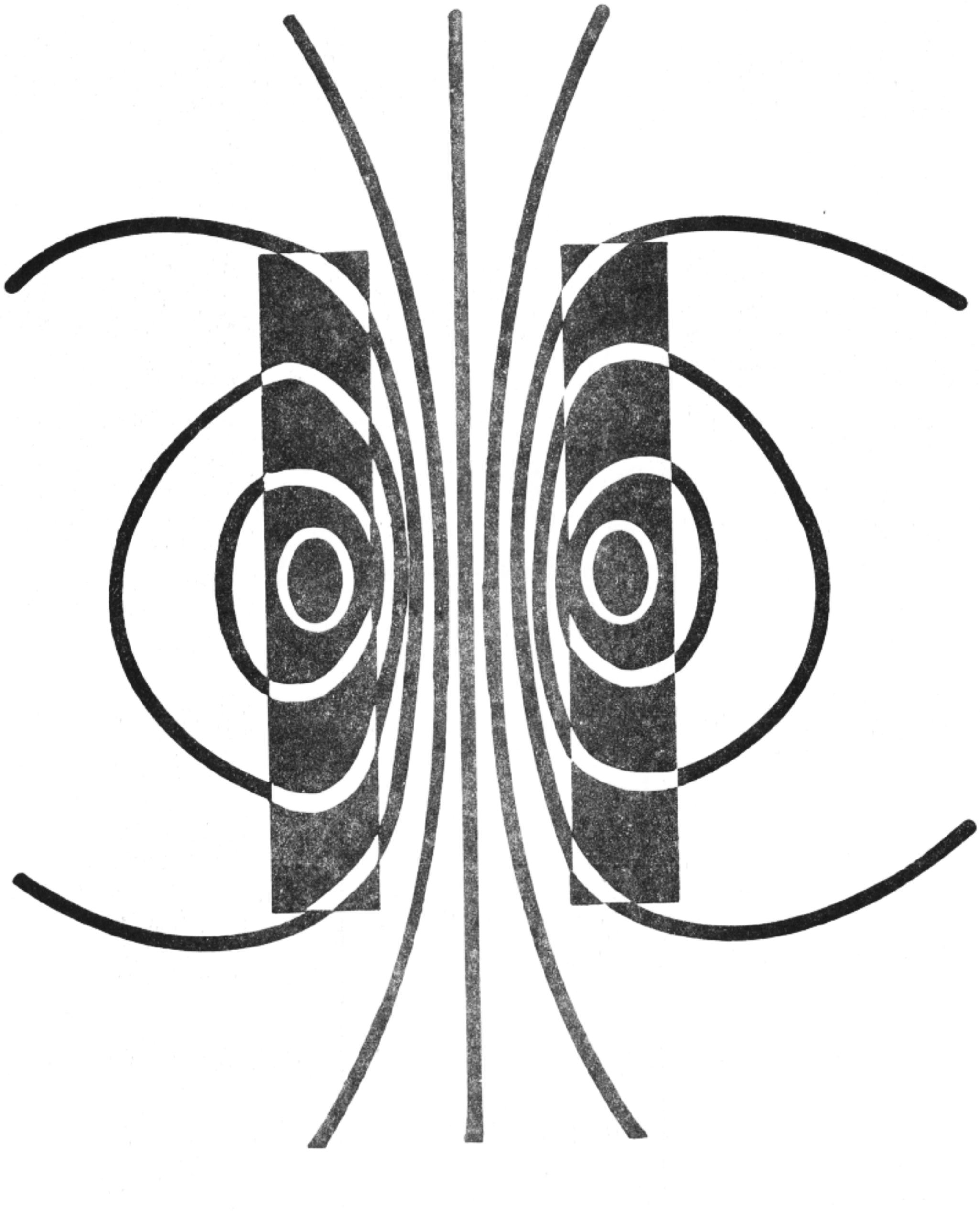

FIGURE 3. BUSS BAR MAGNETIC FIELD

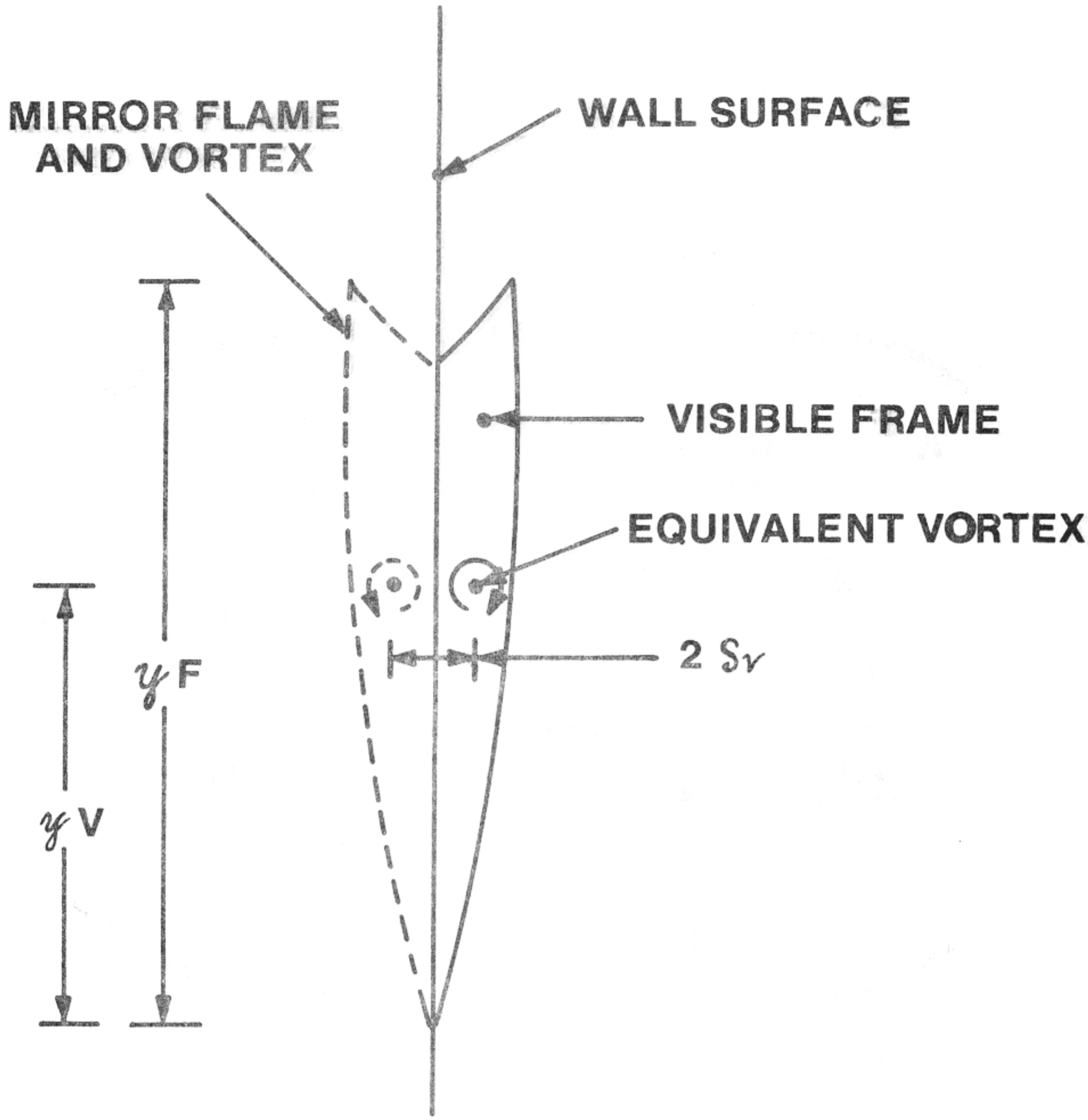

FIGURE 4. FLAME VORTEX REPRESENTATION